

On a possible regularity connecting physical characteristics of a Solar system planet and elements of its orbit.

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### Abstract

This paper is an attempt to detect correlation between characteristics of a big planet of the Solar System (such as mass  $m$ , radius  $r$ , and sidereal period of rotation on its axis  $t$ ) and elements of its orbit (such as radius of a big half-axis of the orbit  $R$  and sidereal period  $T$  of rotation of the planet on the Sun). The existence of this correlation can be generally considered a generalization of the third Kepler's law, which is the logical conclusion of the nebular model of formation of the big planets of the Solar System. There has been made an attempt to find out if this correlation is typical of the big planets with a large number of satellites. The aim of the paper is to search for the stated correlation.

## 1 Search for regularity

It is well known [1] that the third Kepler's law gives the following ratio:

$$\frac{R^3}{T^2} = \text{const.} \quad (1)$$

In point of fact, this ratio demonstrates the following stationary dependence:

$$f(R, T) = \text{const}, \quad (2)$$

There hasnt been any further attempt to improve the law since its discovery because the system of equations in the two-body problem has a completed

solution. However, if we stick to the nebular theory of the origin of the Solar System planets, and consider dynamics, we can admit natural existence of even more universal regularity. It must connect such characteristics of a big planet as mass, radius and period of rotation on its axis with elements of its orbit, such as sidereal period of rotation of the planet on the sun and the big half-axis of its orbit. In this case we can consider regularity (2) to be a particular case of universal regularity. The presence of the latter is the logical consequence of the fact that formation of both big and small planets of the Solar System was affected by the same physical laws. This can be confirmed indirectly by Titsius-Bode law [2]. As is known, the latter establishes strong regularity between the disposition of a given planet in a planetary system and distance between the planet and the sun. This law still doesn't have a satisfactory physical explanation.

Considering the above-stated, and by analogy with expression (2), the desired regularity can be implicitly written as follows:

$$f(m, r, R, T, t) = \text{const}, \quad (3)$$

or, according to Kepler's third law, it can take the following explicit form:

$$m^S r^J R^N T^K t^L = \text{const}. \quad (4)$$

In astronomy the units for measuring the characteristics of planets are the characteristics of the Earth, so if the desired regularity can be unambiguously defined by  $m$ ,  $r$ ,  $R$ ,  $T$ , and  $t$  parameters, ratio 4 for the  $i$  planet of the Solar System can be represented as following:

$$m_i^S r_i^J R_i^N T_i^K t_i^L = 1, \quad (5)$$

where  $S$ ,  $J$ ,  $N$ ,  $K$  and  $L$  exponents are whole numbers common for all planets of the Solar System, but their values are still to be detected. We confine ourselves to the search for values in a rather narrow interval of whole numbers, both positive and negative.

We suppose that  $m$  and  $r$  values for the first planet from expression (5), have some errors, so, in order to find the desired regularity and  $S$ ,  $J$ ,  $N$ ,  $K$  and  $L$  exponents, we can use minimizing functional of the following kind:

$$I(m, r, R, T, t) = \min_{S, J, N, K, L} \max \alpha_i, \quad (6)$$

where

$$\alpha_i = m_i^S r_i^J R_i^N T_i^K t_i^L. \quad (7)$$

We suppose that whole numbers are the most proper to the exponents in the latter ratio. Its obvious that in this case the acceptable solution for the interval of whole numbers from -5 to +5 is the following dependence:

$$\alpha = \frac{m}{r^3} \frac{R^5}{T^3}, \quad (8)$$

The research has shown that there are no other solutions to the concerned task. It follows from the last ratio that the found regularity doesnt include the parameter raised to whole powers. This parameter is obviously connected with the angular moment of the planet.

As long as ratio  $\frac{m}{r^3}$  has density fractal, and on account of expression (1), ratio (8) is equivalent to:

$$\alpha = \rho \sqrt[3]{T} = const, \quad (9)$$

or, using the two planets we can write the following

$$\frac{\rho_i}{\rho_j} = \sqrt[3]{\frac{T_j}{T_i}} \quad (10)$$

In the dimensional system that we have accepted for the Earth ratio (9) converts into equality  $\alpha_{\oplus} = \rho_{\oplus} \sqrt[3]{T_{\oplus}} \equiv 1$ , which allows us to explicitly represent expression (9) in the same dimensional system

$$\alpha_i = \rho_i T_i^{\beta_i} = 1. \quad (11)$$

As we can see  $\beta_{\oplus}$ , exponent for the Earth has value equal to  $\frac{1}{3}$ . In order to find  $\beta_{\oplus}$  exponent value for other objects of the Solar System, which might be different from  $\frac{1}{3}$ , we use the following ratio

$$\beta_i = \beta(i) = \alpha_0 + \alpha_1 i^2 + \alpha_2 i^4 + \alpha_4 i^6, \quad (12)$$

$\alpha_i$  coefficients in this ratio are subject to determination. Using the data on the big planets of the Solar System from the monograph by [3] for the given  $\beta_i$  and  $T_i$  values, and the method of the least squares, we have come to the following approximated ratio of  $\beta_{\oplus}$  values corresponding to the condition  $\alpha_{\oplus} = \rho_{\oplus}^3 \sqrt{T_{\oplus}} \equiv 1$

$$\beta_i = 0.581 - 0.0176i^2 - 0.0146i^4 + 8.37 \cdot 10^{-4}i^6. \quad (13)$$

In the accepted system of numeration for the big planets of the Solar System we have:  $i = -4$  for Mercury,  $i = -3$  for Venus,  $i = -2$  for Earth,  $i = -1$  for Mars  $i = 0$  for Jupiter,  $i = 1$  for Saturn,  $i = 2$  for Uranus,  $i = 3$  for Neptune and  $i = 4$  for Pluto.

## 2 Results

Let us turn directly to the results of our calculations. Table 1 represents values  $\Pi_i = \sqrt{\frac{8\pi^3 G}{c^5} \frac{m R^5}{(r T)^3}}$  (the second column) for the big planets of the Solar System, calculated on account of ratio (8), and their deviations from value, according to the formula  $\Delta\Pi_i = \frac{\Pi_i - \Pi_0}{\Pi_0} (i = 1, 2, 3 \dots 9)$  given as fractions of  $\Pi_0$  value, where  $\Pi_0$  value itself is referred to the Earth, which is a starting point for counting out. In order to find them we use the data on the big planets of the Solar System, represented in the monograph by [3]. Table 2 contains the same  $\Pi_0$  values for 16 satellites of Jupiter, and their  $\Delta\Pi_i$  deviations calculated according to the foregoing formula. We took the mean value for all satellites

$\sum_{i=1}^{12} \Pi_i$  as value, that is  $\frac{i=1}{N}$  value which equals  $8.1262 \cdot 10^{-10}$  to in case  $N=16$ . Analogous calculations have been done for the satellites of Saturn, Uranus and Neptune. The results appear to be similar to those given in Table 2. For this

reason we limited ourselves to demonstration of the results for the planetary system of Jupiter only. In all cases we borrowed the data on satellites of the stated planets from the Internet (<http://ggreen.chat.ru/supiter.html>).

*Big planets*

	$\Pi_i$	$\Delta\Pi_i$
<i>Mercury</i>	$4.82482 \cdot 10^{-7}$	-0.214
<i>Venus</i>	$5.44790 \cdot 10^{-7}$	-0.099
<i>Earth</i>	$6.14015 \cdot 10^{-7}$	0
<i>Mars</i>	$5.76384 \cdot 10^{-7}$	-0.061
<i>Jupiter</i>	$4.47971 \cdot 10^{-7}$	-0.282
<i>Saturn</i>	$3.70446 \cdot 10^{-7}$	-0.397
<i>Uranus</i>	$6.13666 \cdot 10^{-7}$	-0.004
<i>Neptune</i>	$7.87440 \cdot 10^{-7}$	+0.267
<i>Pluto</i>	$6.74395 \cdot 10^{-7}$	+0.100

Table 1

First of all, let us turn to the analysis of the Table 1 data. Of course we can't expect that the universal law of type (3) has remained invariable for the whole period of the Solar System existence. It is more reasonable to suppose that it has evolved under the influence of different factors [4]. In the first place among them is the influence of fluxes from the Sun itself. But for all that, the farther the planet is from the Sun, the less is the fluxes effect on it. It appears that in this respect Mercury and Venus have been in the zone of the biggest flux influence of the central heavenly body for the whole period of their existence. This, apparently, can explain appearance of big  $\Pi_0$  values of such planets as Mercury, Jupiter and Saturn. There are no doubts that the giant planets (Jupiter and Saturn), which are much bigger in mass than other planets of the Solar System, have substantially influenced the characteristics of the orbits especially those of the closely disposed planets. This, apparently, can explain the presence of high  $\Pi_0$  values for Neptune.

Along with flux processes connected with the Sun and giant planets, the orbits of the big planets could also evolve under the influence of various dissipative powers of different nature. For these reasons even if there was the law of type (3) in the distant past, it has had certain (perhaps, significant)

changes, but has still lasted out, though with slight deviations from the original form. It can explain the appearance of big deviations  $\Delta\Pi_i$  for the big planets of the Solar System, which are given in Table 1. Thus, it is quite safe to claim that type (3) regularity exists as regularity that appeared in the period of the Solar System formation, since  $\Delta\Pi_i$  values haven't changed much for the time of its existence, and on the whole, are far less than 1 (they do not exceed 0.4).

*Satellites of Jupiter*

	$\Pi_i$	$\Delta\Pi_i$	
<i>Metis</i>	4.10316 $10^{-10}$	-0.495	
<i>Adrastea</i>	5.25251 $10^{-10}$	-0.353	
<i>Amaltea</i>	1.15788 $10^{-10}$	-0.858	
<i>Thebe</i>	1.31448 $10^{-10}$	-0.838	
<i>Io</i>	2.06784 $10^{-10}$	-0.745	
<i>Europa</i>	2.34124 $10^{-10}$	-0.343	
<i>Ganimede</i>	5.73522 $10^{-10}$	-0.294	
<i>Callisto</i>	6.48570 $10^{-10}$	-0.202	(14)
<i>Leda</i>	1.21020 $10^{-10}$	-0.851	
<i>Himalia</i>	12.60 $10^{-10}$	+0.550	
<i>Lysithea</i>	13.3251 $10^{-10}$	+0.640	
<i>Elara</i>	13.7793 $10^{-10}$	+0.696	
<i>Ananke</i>	14.7848 $10^{-10}$	+0.819	
<i>Carme</i>	15.0074 $10^{-10}$	+0.847	
<i>Pasiphae</i>	15.2882 $10^{-10}$	+0.881	
<i>Sinope</i>	15.5661 $10^{-10}$	+0.916	

Table 2

However, it is difficult to be certain about the existence of regularity of type (3) when we deal with the satellites of the big planets. In this case the master of the situation is seemingly the planet forming its own satellite system. That is why the light deviations from mean  $\Delta\Pi_i$  values, about 0.2-0.5, represented in Table 2 can be first of all explained by errors in determination of characteristics of the satellites of the planetary system, but not by some other factors.

## References

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